

Figure 2.1 Data objects in Prolog.

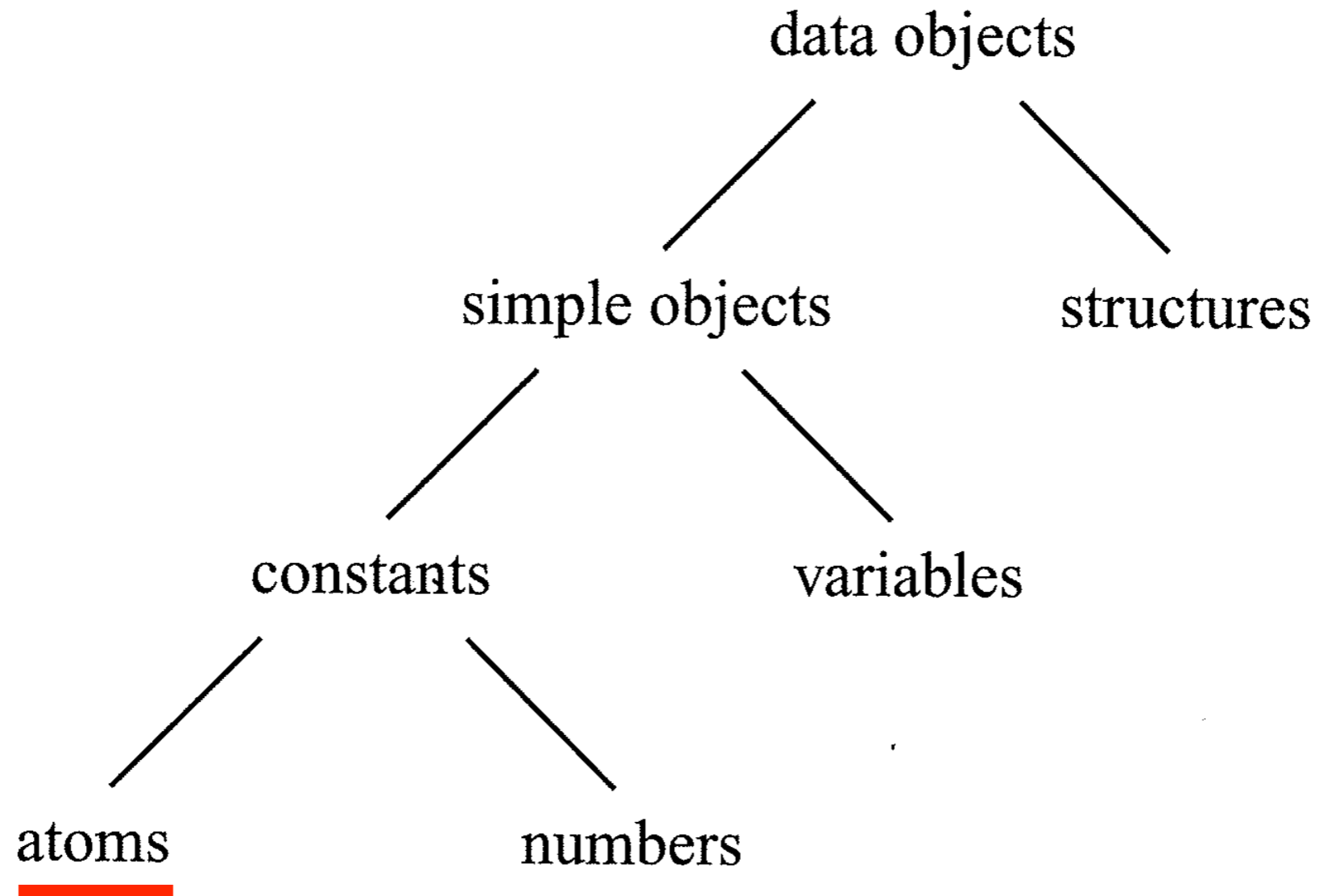


Figure 2.1 Data objects in Prolog.

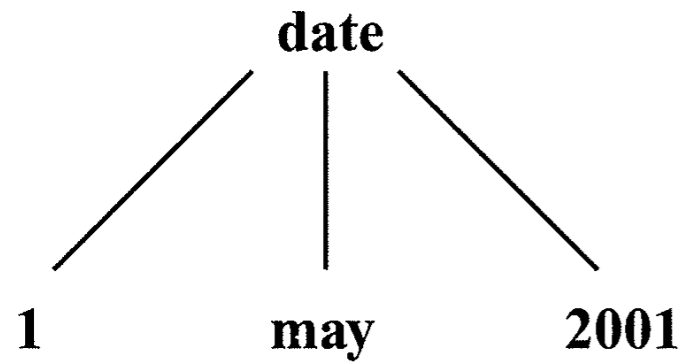
Atoms

- Start with lowercase letters
- Strings of special characters, e.g. :- is an atom
- Enclosed in single quotes, e.g. 'Tom' is an atom

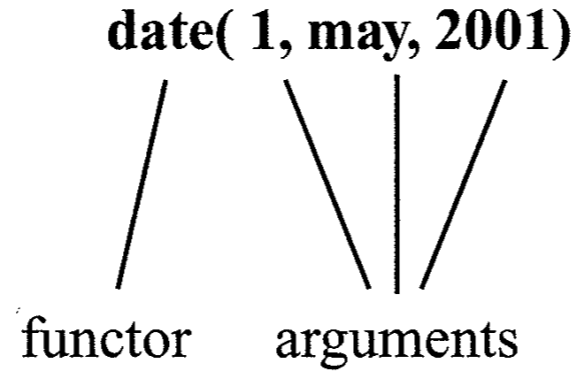
Anonymous variables

- Singleton variable – variable in a rule that is named but not used
- Anonymous variable – an unnamed variable in a rule
- Avoid singleton variables

Structures



(a)



(b)

Figure 2.2 Date is an example of a structured object: (a) as it is represented as a tree; (b) as it is written in Prolog.

Terms

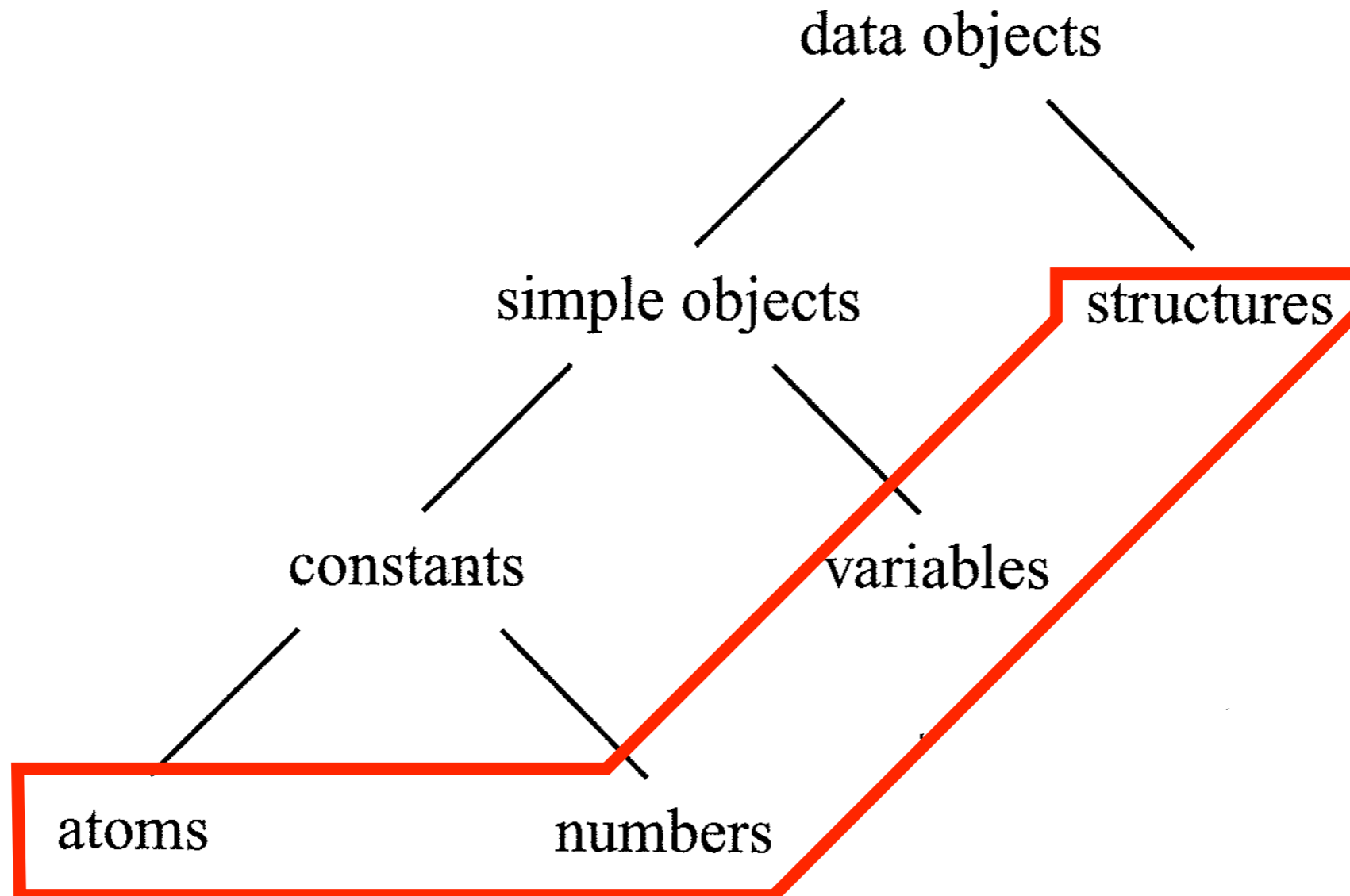


Figure 2.1 Data objects in Prolog.

Atoms, numbers, variables and structures are all terms.

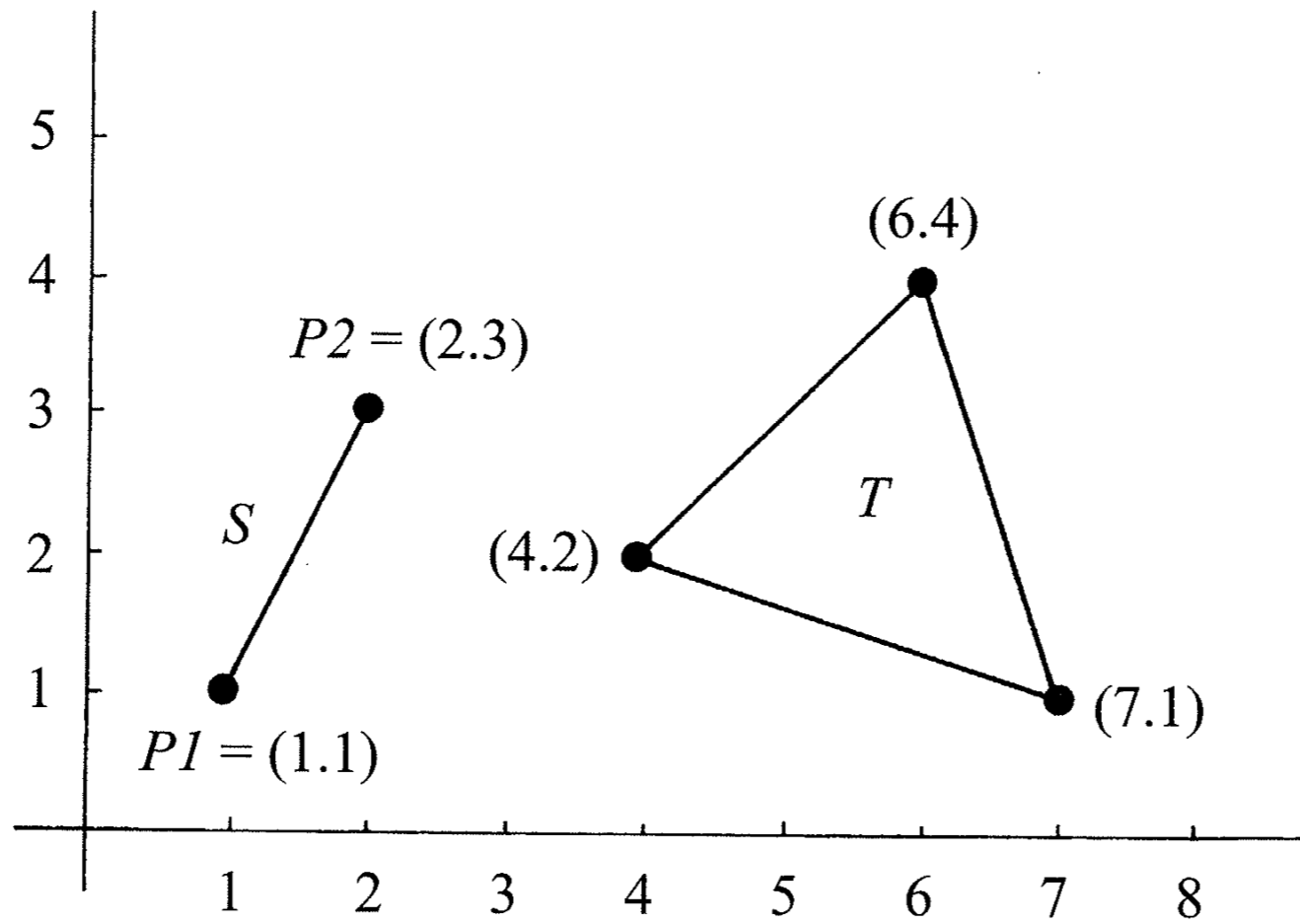


Figure 2.3 Some simple geometric objects.

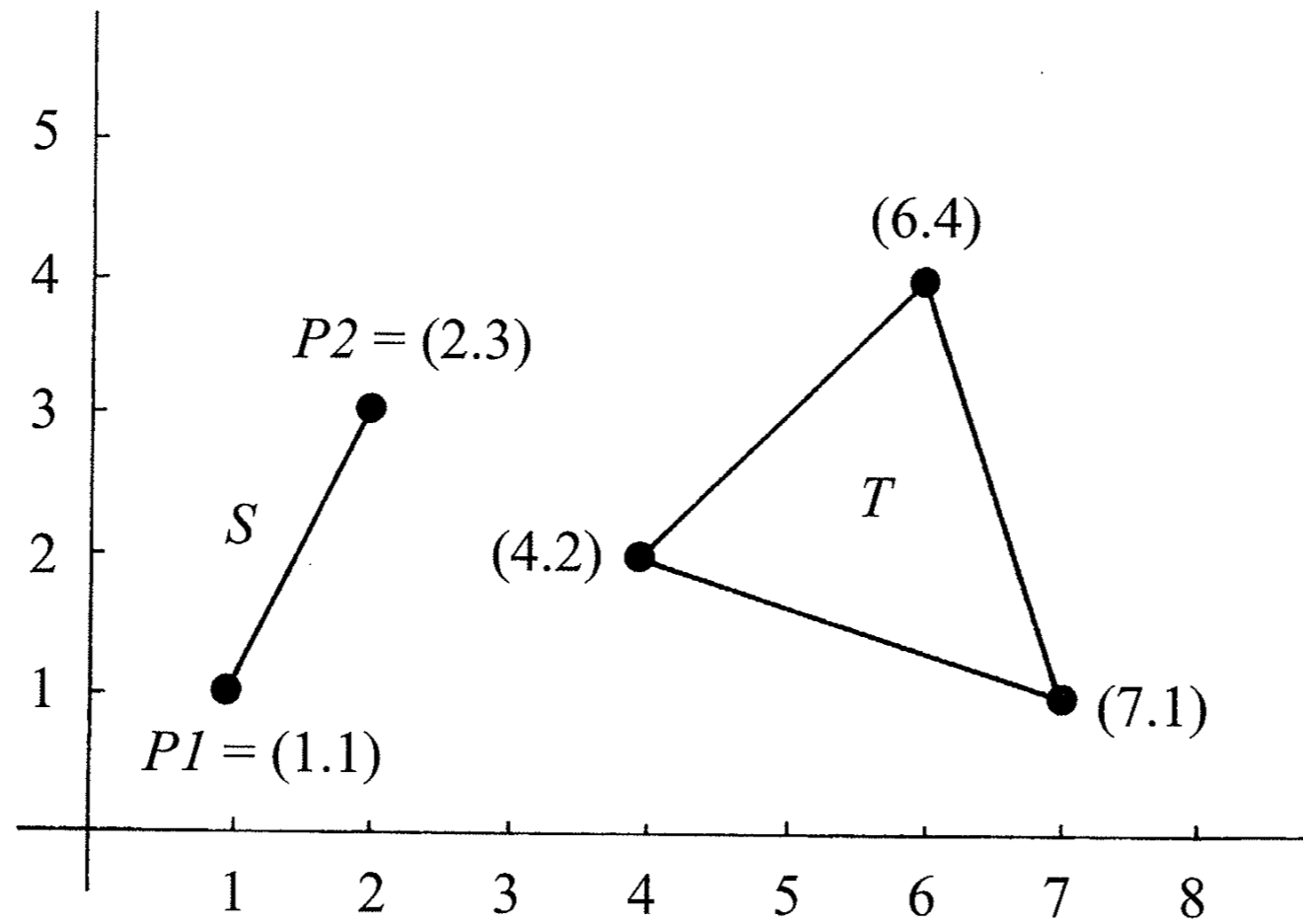


Figure 2.3 Some simple geometric objects.

P1 represented as `point(1, 1)`.

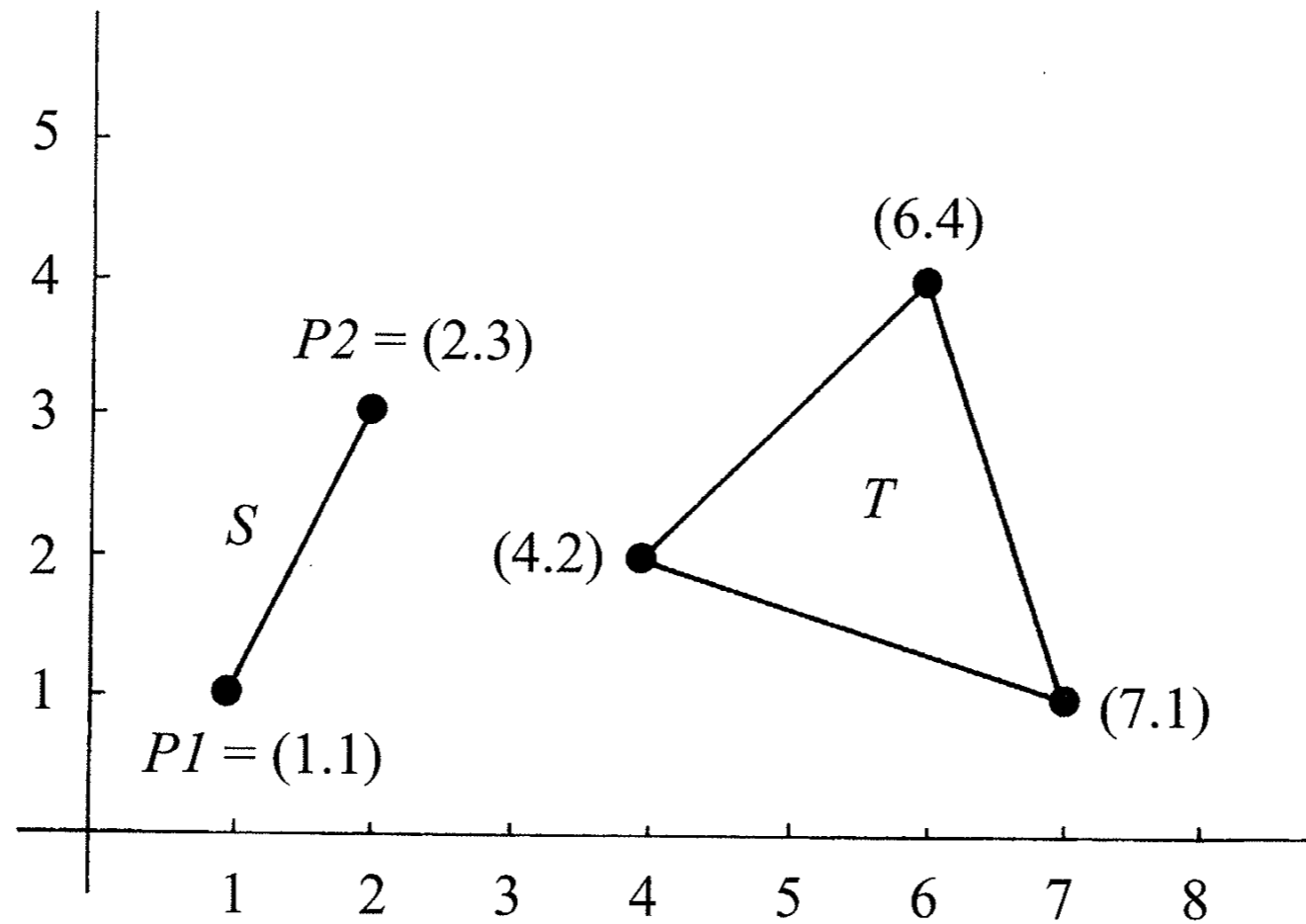


Figure 2.3 Some simple geometric objects.

$P1$ represented as `point(1, 1)`.

S represented as `seg(point(1, 1), point(2, 3)`.

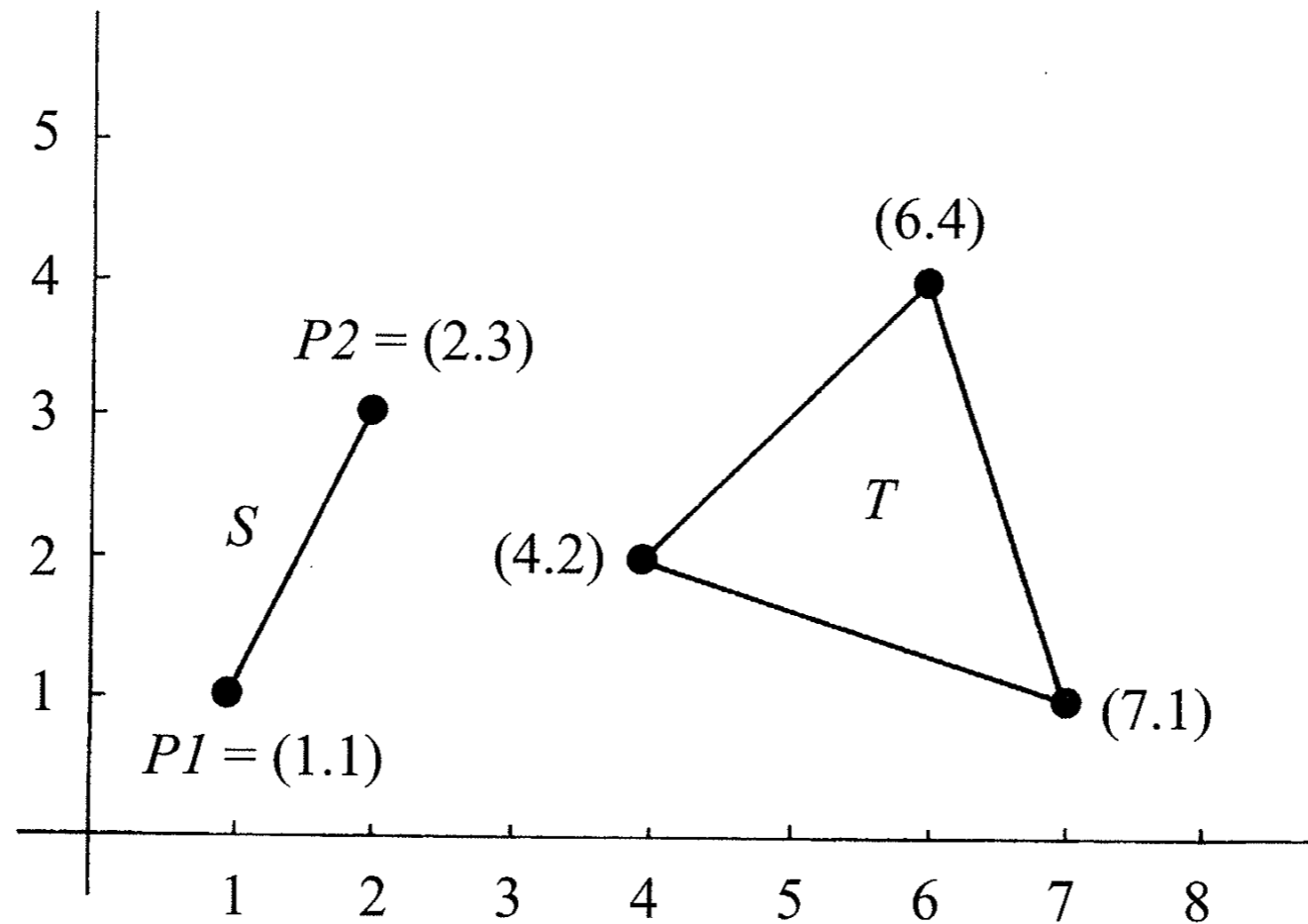


Figure 2.3 Some simple geometric objects.

$P1$ represented as `point(1, 1)`.

S represented as `seg(point(1, 1), point(2, 3))`.

T represented as

`triangle(point(4, 2), point(6, 4), point(7, 1))`.

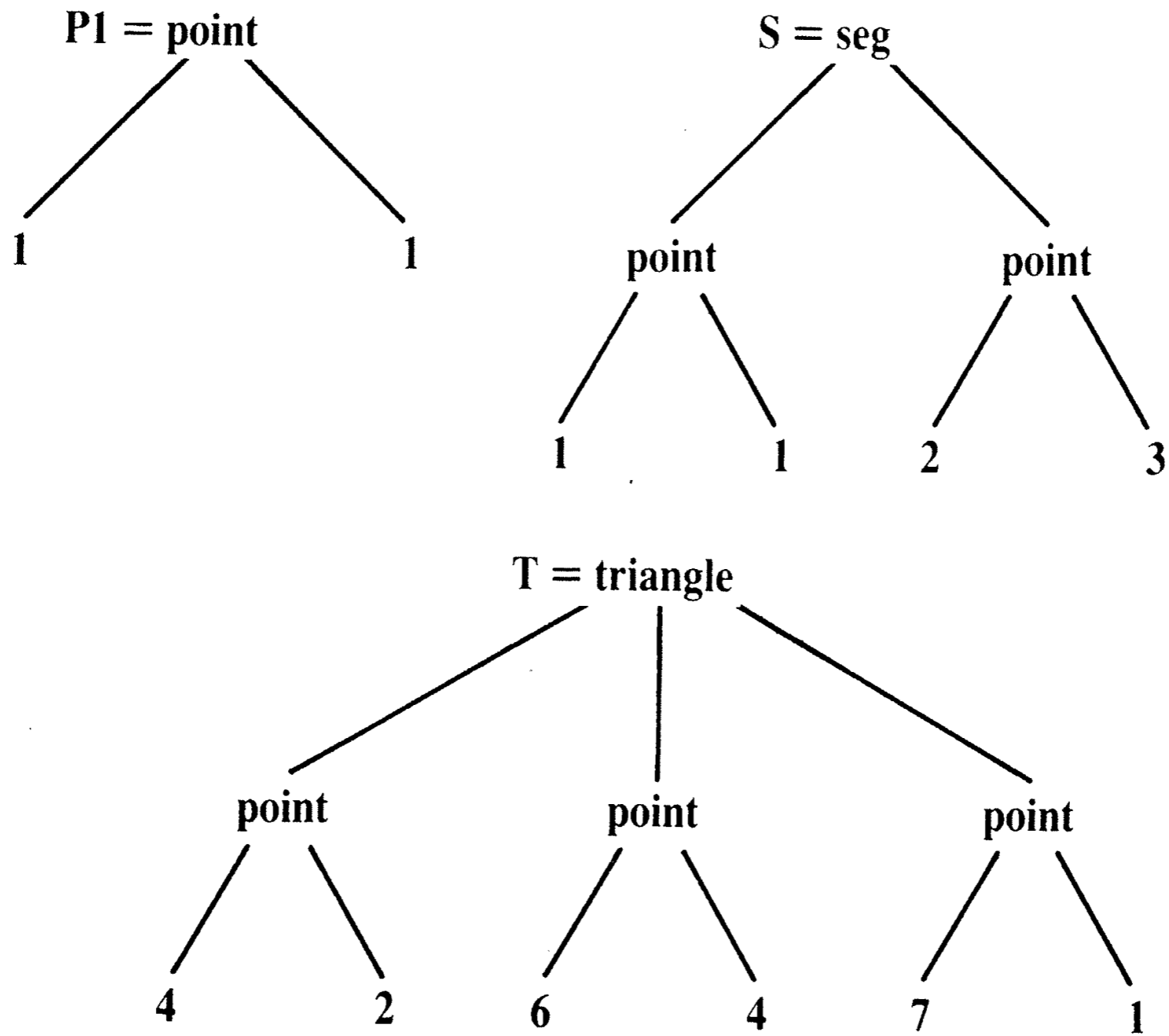


Figure 2.4 Tree representation of the objects in Figure 2.3.

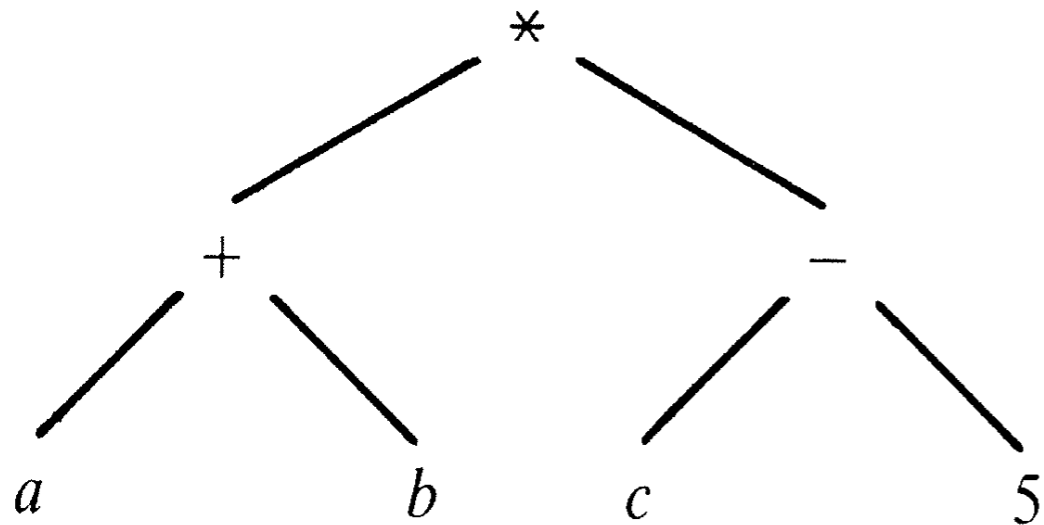


Figure 2.5 A tree structure that corresponds to the arithmetic expression $(a + b) * (c - 5)$.

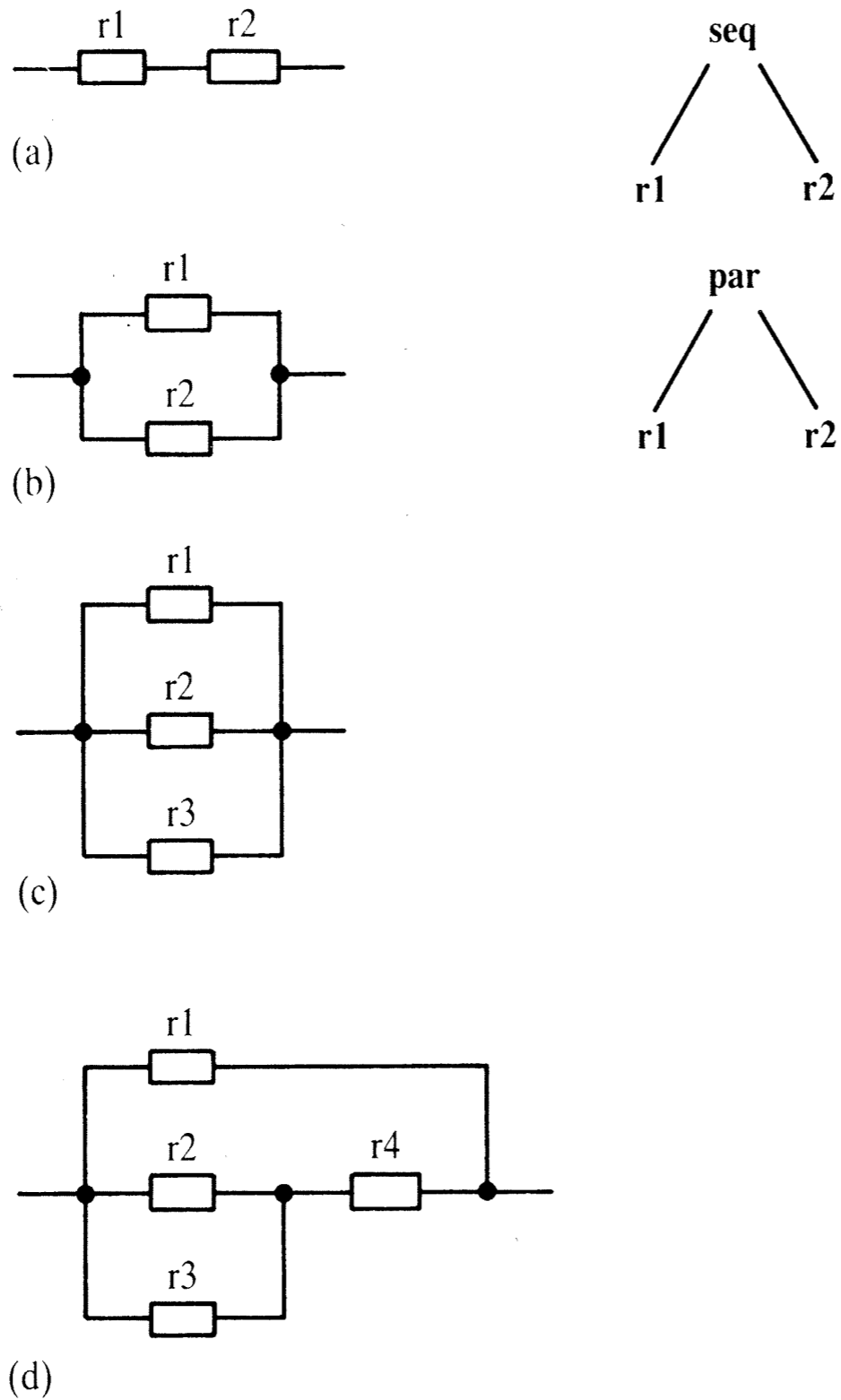


Figure 2.6 Some simple electric circuits and their tree representations: (a) sequential composition of resistors r_1 and r_2 ; (b) parallel composition of two resistors; (c) parallel composition of three resistors; (d) parallel composition of r_1 and another circuit.

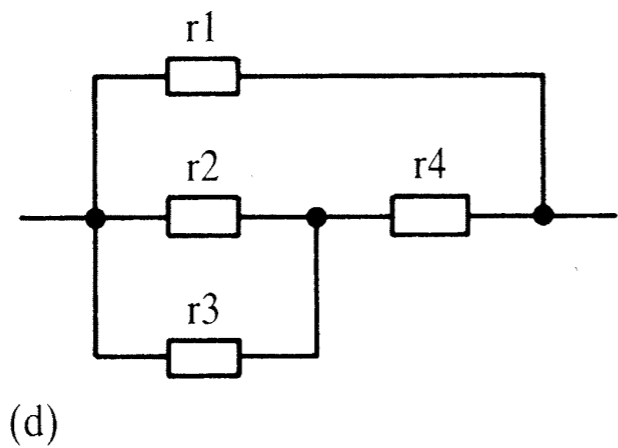
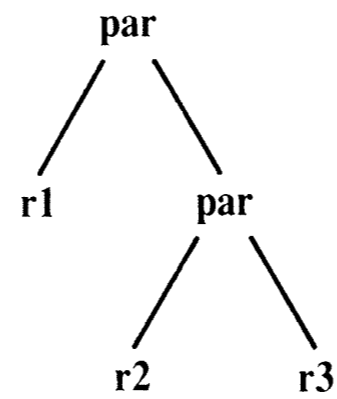
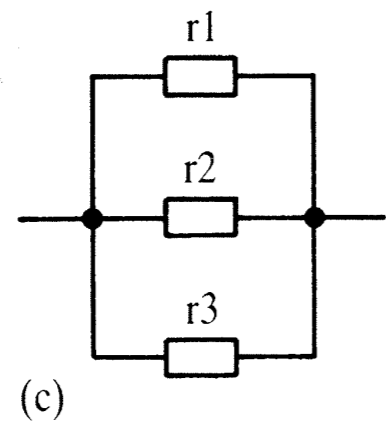
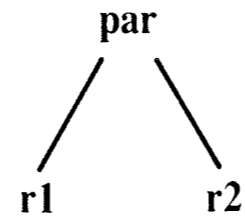
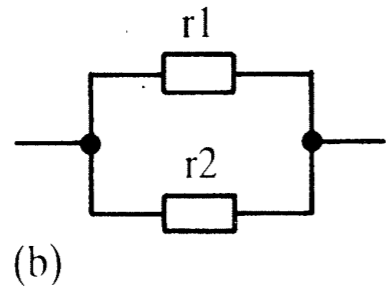
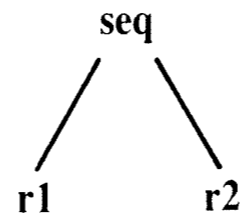
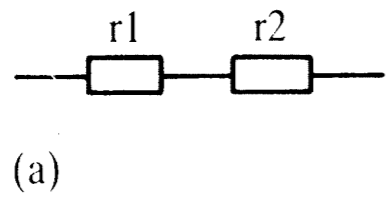


Figure 2.6 Some simple electric circuits and their tree representations: (a) sequential composition of resistors r_1 and r_2 ; (b) parallel composition of two resistors; (c) parallel composition of three resistors; (d) parallel composition of r_1 and another circuit.

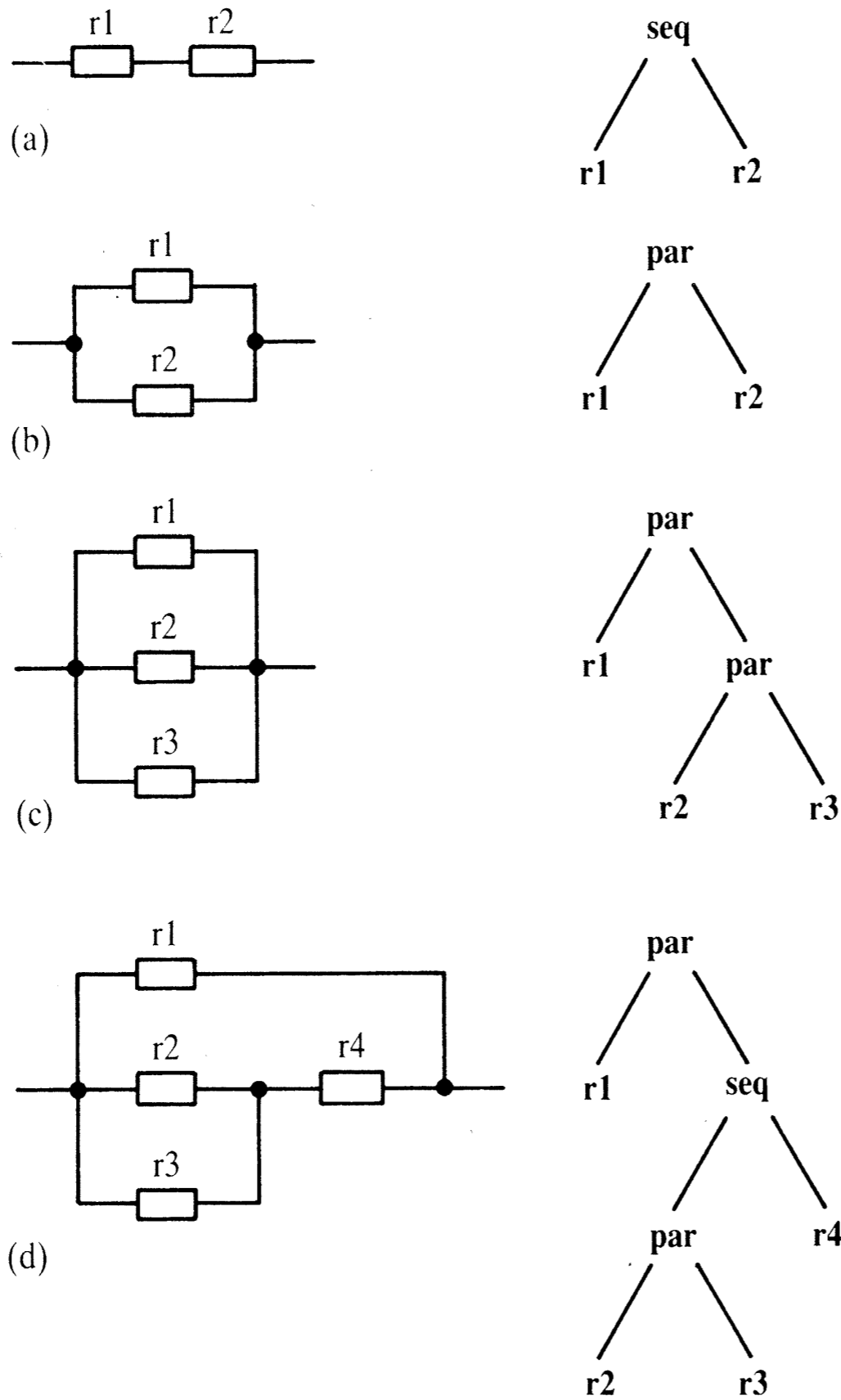


Figure 2.6 Some simple electric circuits and their tree representations: (a) sequential composition of resistors $r1$ and $r2$; (b) parallel composition of two resistors; (c) parallel composition of three resistors; (d) parallel composition of $r1$ and another circuit.

Matching

= is the matching operator

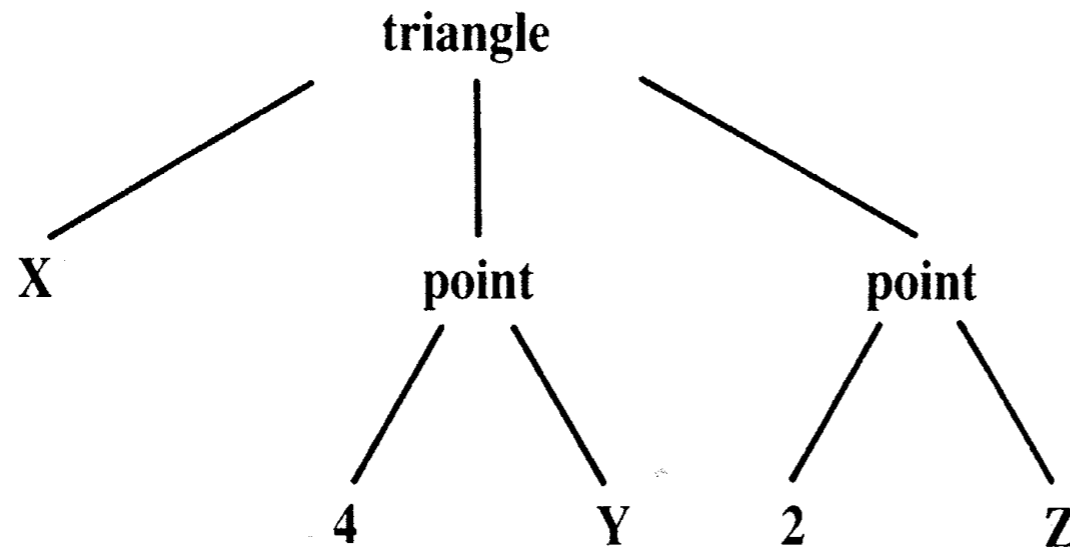
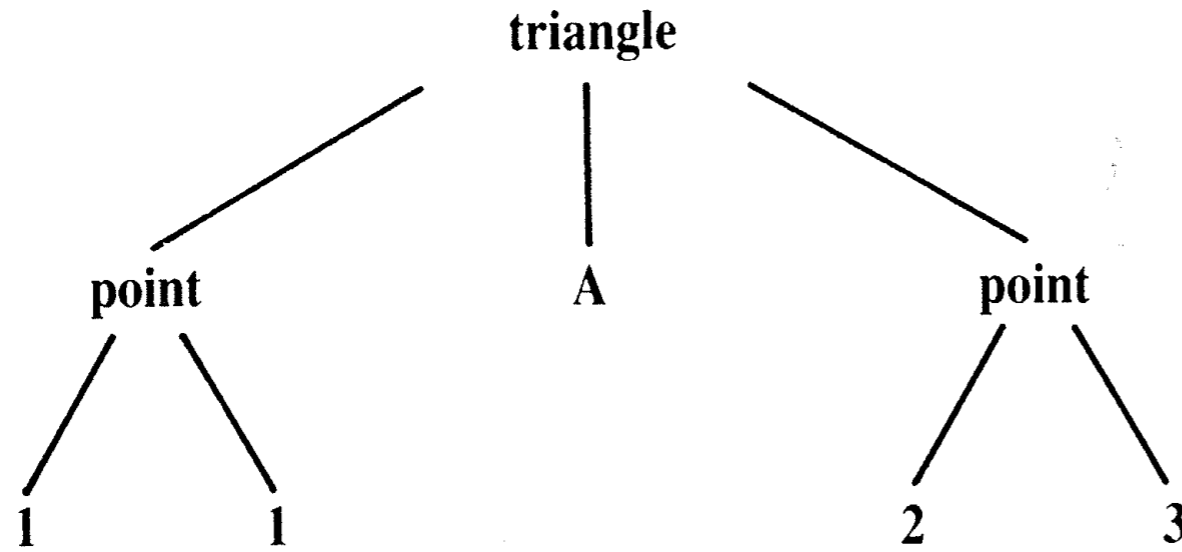


Figure 2.7 Matching $\text{triangle}(\text{point}(1,1), A, \text{point}(2,3)) = \text{triangle}(X, \text{point}(4,Y), \text{point}(2,Z))$

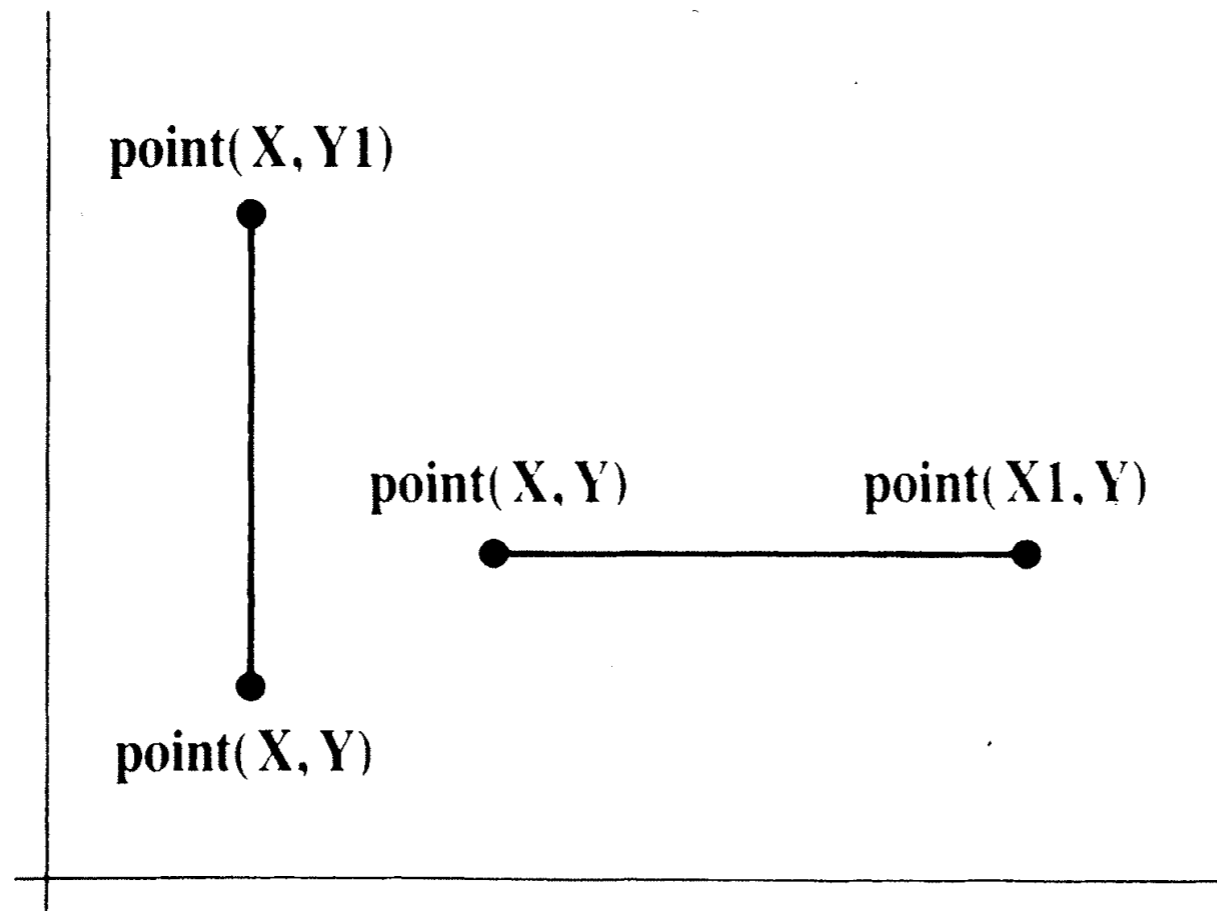


Figure 2.8 Illustration of vertical and horizontal line segments.

Matching

= is the matching operator

- (1) If S and T are constants then S and T match only if they are the same object.
- (2) If S is a variable and T is anything, then they match, and S is instantiated to T . Conversely, if T is a variable then T is instantiated to S .
- (3) If S and T are structures then they match only if
 - (a) S and T have the same principal functor, and
 - (b) all their corresponding components match.

The resulting instantiation is determined by the matching of the components.

Definitions

Clause – a rule or a fact

Instance of clause c – c with each variable substituted by some term.

, is conjunction “and”

; is disjunction “or”

$P :- Q ; R.$

is the same as

$P :- Q.$

$P :- R.$

Declarative meaning – what?

A goal G is true (that is, satisfiable, or logically follows from the program) if and only if:

- (1) there is a clause C in the program such that
- (2) there is a clause instance I of C such that
 - (a) the head of I is identical to G , and
 - (b) all the goals in the body of I are true.

Query, ?- G .

head body
 G :- Q, R, S .
 ↑ ↑ ↑
 goals

Procedural meaning – how?

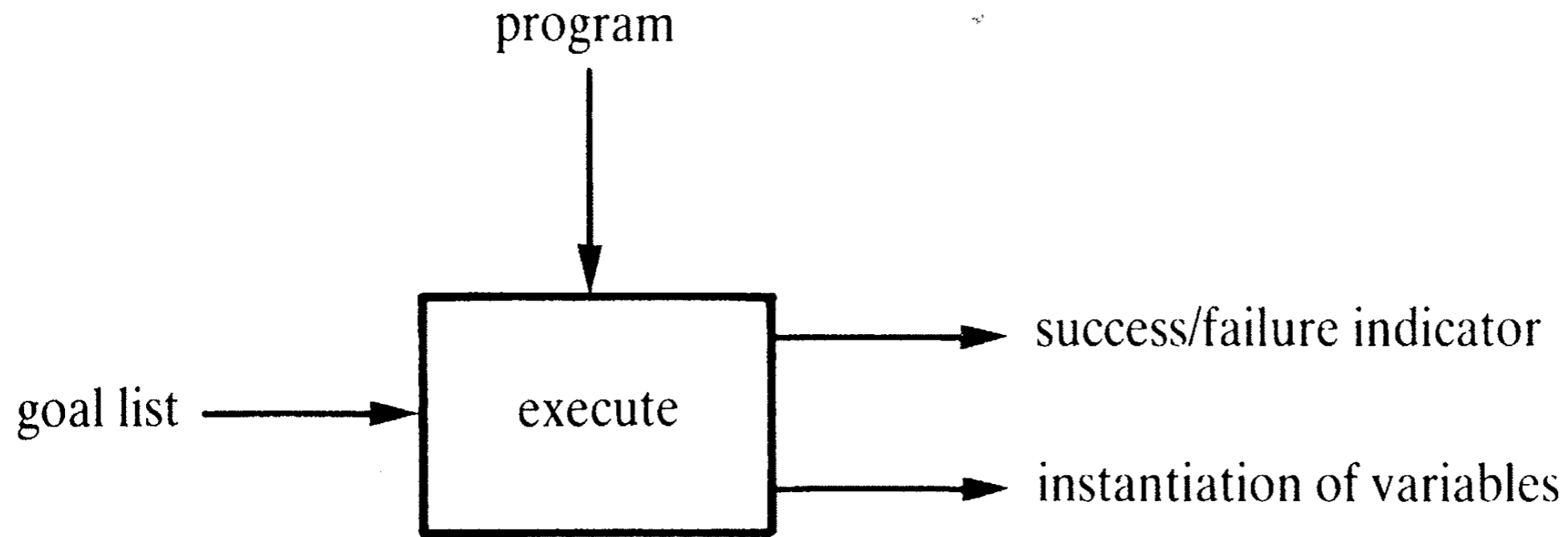


Figure 2.9 Input/output view of the procedure that executes a list of goals.

Program (data base)

```
→ big( bear).           % Clause 1
big( elephant).        % Clause 2
small( cat).           % Clause 3

brown( bear).          % Clause 4
black( cat).           % Clause 5
gray( elephant).      % Clause 6
dark( Z) :-
    black( Z).         % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).         % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
dark( X), big( X).
```

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3

brown( bear).        % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
dark( X), big( X).
```

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3

brown( bear).        % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
→ dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
black( X), big( X).
```


Program (data base)

```
→ big( bear).           % Clause 1
big( elephant).        % Clause 2
small( cat).           % Clause 3

brown( bear).          % Clause 4
black( cat).           % Clause 5
gray( elephant).      % Clause 6
dark( Z) :-
    black( Z).         % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).         % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
black( X), big( X).
```

Program (data base)

```
big( bear).      % Clause 1
big( elephant). % Clause 2
small( cat).     % Clause 3

brown( bear).    % Clause 4
black( cat).     % Clause 5
gray( elephant). % Clause 6
dark( Z) :-
    black( Z).   % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).   % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
black( X), big( X).
```

Program (data base)

```
big( bear).          % Clause 1
big( elephant).     % Clause 2
small( cat).        % Clause 3

brown( bear).       % Clause 4
black( cat).        % Clause 5
gray( elephant).   % Clause 6
dark( Z) :-
    black( Z).      % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).      % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
big( cat).
```

Program (data base)

```
→ big( bear).           % Clause 1
big( elephant).        % Clause 2
small( cat).           % Clause 3

brown( bear).          % Clause 4
black( cat).           % Clause 5
gray( elephant).      % Clause 6
dark( Z) :-
    black( Z).         % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).         % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
big( cat).
```

Program (data base)

```
big( bear).          % Clause 1
big( elephant).     % Clause 2
small( cat).        % Clause 3

brown( bear).       % Clause 4
black( cat).        % Clause 5
gray( elephant).   % Clause 6
dark( Z) :-
    black( Z).      % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).      % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```



Execution trace

```
big( cat).
```

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3

brown( bear).        % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

 **Fail**

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
big( cat).
```

Program (data base)

```
big( bear).      % Clause 1
big( elephant). % Clause 2
small( cat).     % Clause 3

brown( bear).   % Clause 4
black( cat).    % Clause 5
gray( elephant). % Clause 6
dark( Z) :-
    black( Z).  % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).  % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
black( X), big( X).
```

Backtrack to black(cat).

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3

brown( bear).        % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
black( X), big( X).
```


Program (data base)

```
big( bear).          % Clause 1
big( elephant).     % Clause 2
small( cat).        % Clause 3

brown( bear).       % Clause 4
black( cat).        % Clause 5
gray( elephant).   % Clause 6
dark( Z) :-
    black( Z).      % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).      % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```



Execution trace

```
black( X), big( X).
```

Program (data base)

```
big( bear).          % Clause 1
big( elephant).     % Clause 2
small( cat).        % Clause 3

brown( bear).       % Clause 4
black( cat).        % Clause 5
gray( elephant).   % Clause 6
dark( Z) :-
    black( Z).      % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).      % Clause 8: Anything brown is dark
```

 **Fail**

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
black( X), big( X).
```

Program (data base)

```
big( bear).      % Clause 1
big( elephant). % Clause 2
small( cat).     % Clause 3

brown( bear).   % Clause 4
black( cat).    % Clause 5
gray( elephant). % Clause 6
dark( Z) :-
    black( Z).  % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).  % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```



Execution trace

```
black( X), big( X).
```

Backtrack to dark(Z) .

Program (data base)

```
big( bear).      % Clause 1
big( elephant). % Clause 2
small( cat).     % Clause 3

brown( bear).   % Clause 4
black( cat).    % Clause 5
gray( elephant). % Clause 6
→ dark( Z) :-
    black( Z).  % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).  % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
dark( X), big( X).
```

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3

brown( bear).        % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
→ dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
dark( X), big( X).
```

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3

brown( bear).        % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
→ dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```


Goal list

```
?- dark( X), big( X).
```

Execution trace

```
brown( X), big( X).
```

Program (data base)

 big(bear). % Clause 1
big(elephant). % Clause 2
small(cat). % Clause 3

brown(bear). % Clause 4
black(cat). % Clause 5
gray(elephant). % Clause 6
dark(Z) :-
 black(Z). % Clause 7: Anything black is dark
dark(Z) :-
 brown(Z). % Clause 8: Anything brown is dark

Goal list

?- dark(X), big(X).

Execution trace

brown(X), big(X).

Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3
→ brown( bear).      % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
brown( X), big( X).
```


Program (data base)

```
big( bear).           % Clause 1
big( elephant).      % Clause 2
small( cat).         % Clause 3
→ brown( bear).      % Clause 4
black( cat).         % Clause 5
gray( elephant).    % Clause 6
dark( Z) :-
    black( Z).       % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).       % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
big( bear).
```

Program (data base)

```
→ big( bear).           % Clause 1
big( elephant).        % Clause 2
small( cat).           % Clause 3

brown( bear).          % Clause 4
black( cat).           % Clause 5
gray( elephant).       % Clause 6
dark( Z) :-
    black( Z).          % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).          % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
big( bear).
```

Program (data base)

```
→ big( bear).           % Clause 1
big( elephant).        % Clause 2
small( cat).           % Clause 3

brown( bear).          % Clause 4
black( cat).           % Clause 5
gray( elephant).      % Clause 6
dark( Z) :-
    black( Z).         % Clause 7: Anything black is dark
dark( Z) :-
    brown( Z).         % Clause 8: Anything brown is dark
```

Goal list

```
?- dark( X), big( X).
```

Execution trace

```
X = bear
```

procedure *execute* (*Program*, *GoalList*, *Success*);

Input arguments:

Program: list of clauses

GoalList: list of goals

Output argument:

Success: truth value; *Success* will become true if *GoalList* is true with respect to *Program*

Local variables:

Goal: goal

OtherGoals: list of goals

Satisfied: truth value

MatchOK: truth value

Instant: instantiation of variables

H, *H'*, *B1*, *B1'*, . . . , *Bn*, *Bn'*: goals

Auxiliary functions:

empty(L): returns true if *L* is the empty list

head(L): returns the first element of list *L*

tail(L): returns the rest of *L*

append(L1,L2): appends list *L2* at the end of list *L1*

match(T1,T2,MatchOK,Instant): tries to match terms *T1* and *T2*; if succeeds

then *MatchOK* is true and *Instant* is the corresponding instantiation of variables

substitute(Instant, Goals): substitutes variables in *Goals* according to instantiation *Instant*

```

begin
  if empty(GoalList) then Success := true
  else
    begin
      Goal := head(GoalList);
      OtherGoals := tail(GoalList);
      Satisfied := false;
      while not Satisfied and "more clauses in program" do
        begin
          Let next clause in Program be
             $H :- B1, \dots, Bn.$ 
          Construct a variant of this clause
             $H' :- B1', \dots, Bn'.$ 
          match(Goal,H',MatchOK,Instant);
          if MatchOK then
            begin
              NewGoals := append([B1', \dots, Bn'], OtherGoals);
              NewGoals := substitute(Instant,NewGoals);
              execute(Program,NewGoals,Satisfied)
            end
          end;
          Success := Satisfied
        end
      end;
    end
  end;
end;

```

Reordering clauses and goals

- Reordering can have a big effect on efficiency.
- In extreme cases, reordering can cause an infinite recursive loop.

```

anc2( X, Z):-
  parent( X, Y),
  anc2( Y, Z).

anc2( X, Z):-
  parent( X, Z).

```

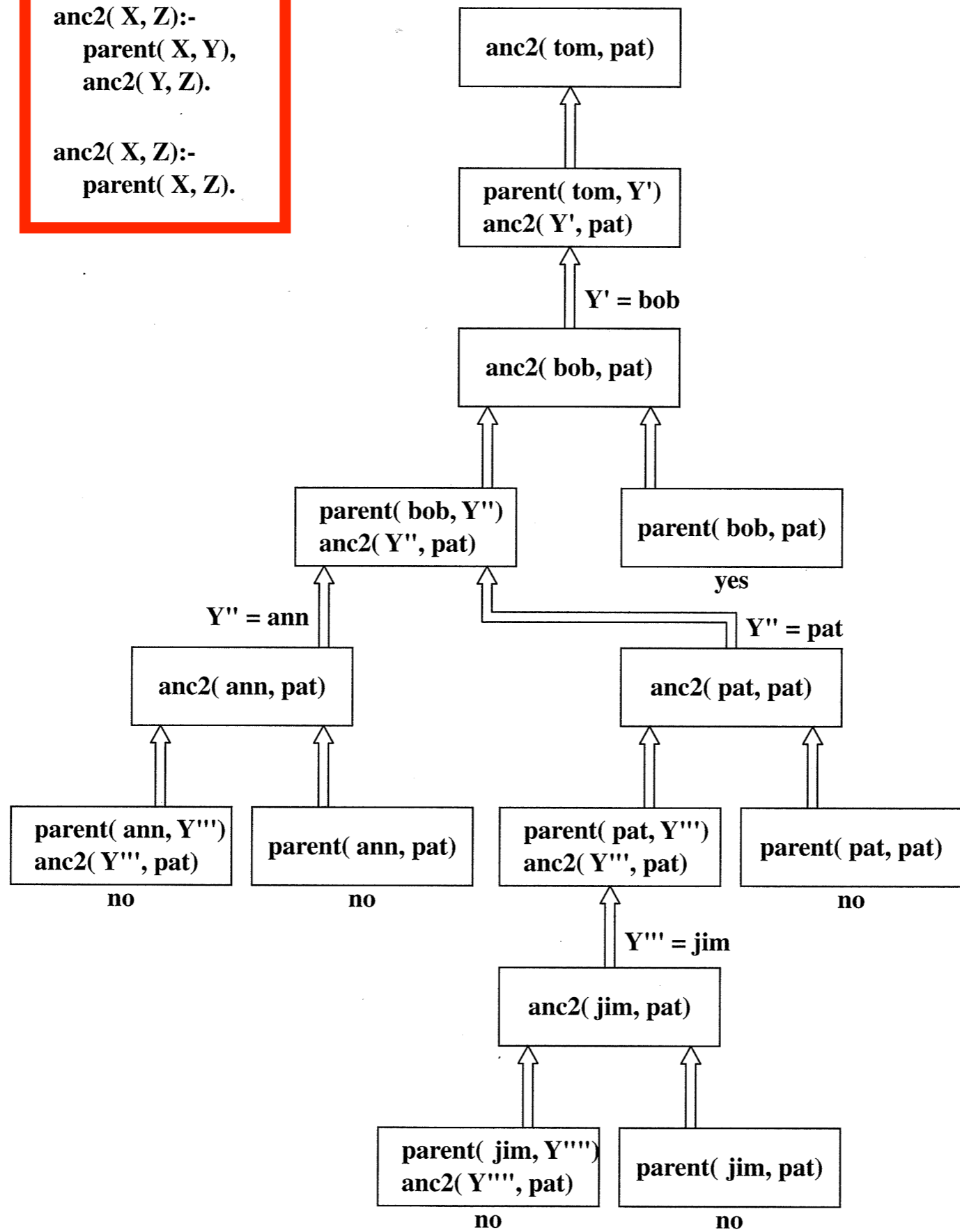


Figure 2.13 The complete execution trace to satisfy the goal `anc2(tom, pat)`. All the alternative paths in the large left subtree fail, before the right-most path succeeds.

```

anc3( X, Z):-
  parent( X, Z).

anc3( X, Z):-
  anc3( X, Y),
  parent( Y, Z).

```

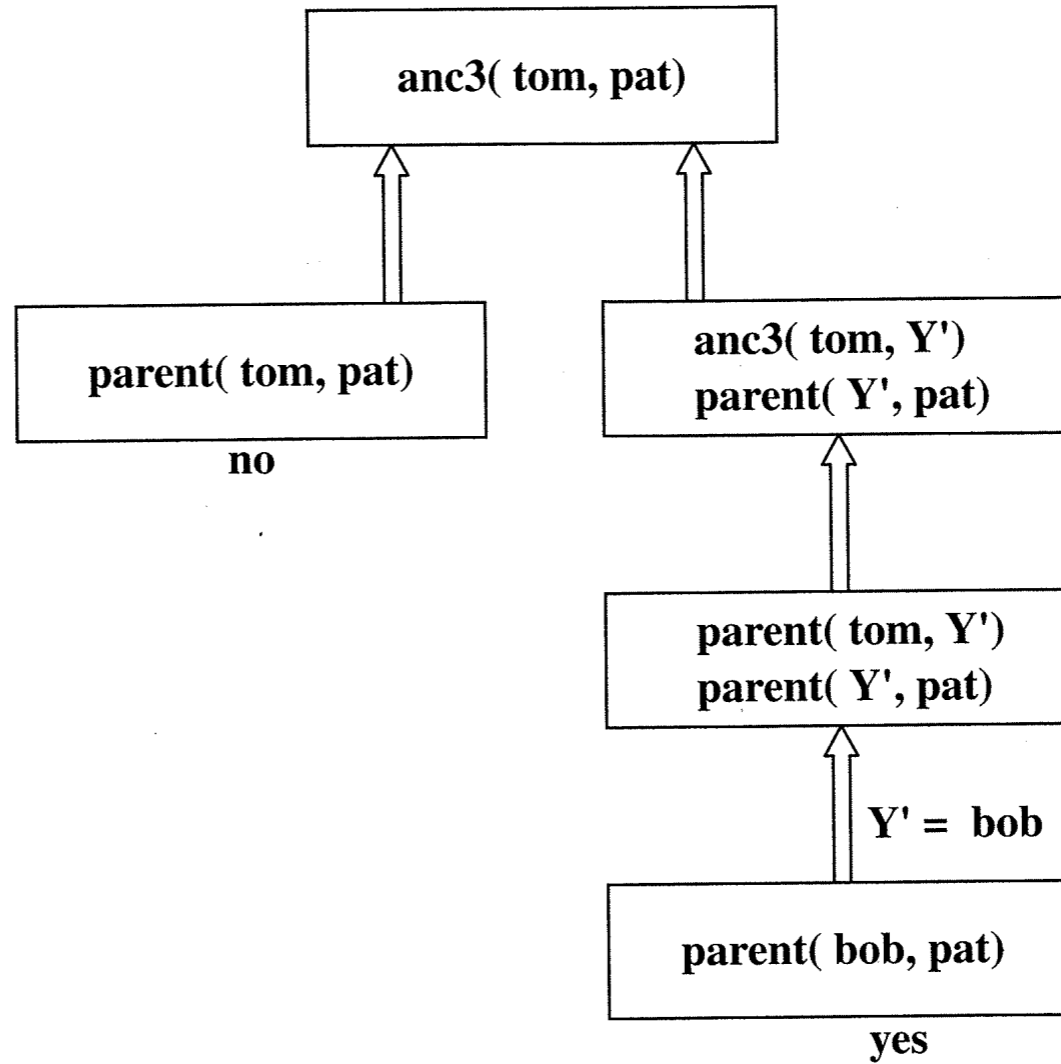


Figure 2.14 The execution trace to satisfy the goal `anc3(tom, pat)`.


```
anc4( X, Z):-  
  anc4( X, Y),  
  parent( Y, Z).  
  
anc4( X, Z):-  
  parent( X, Z).
```

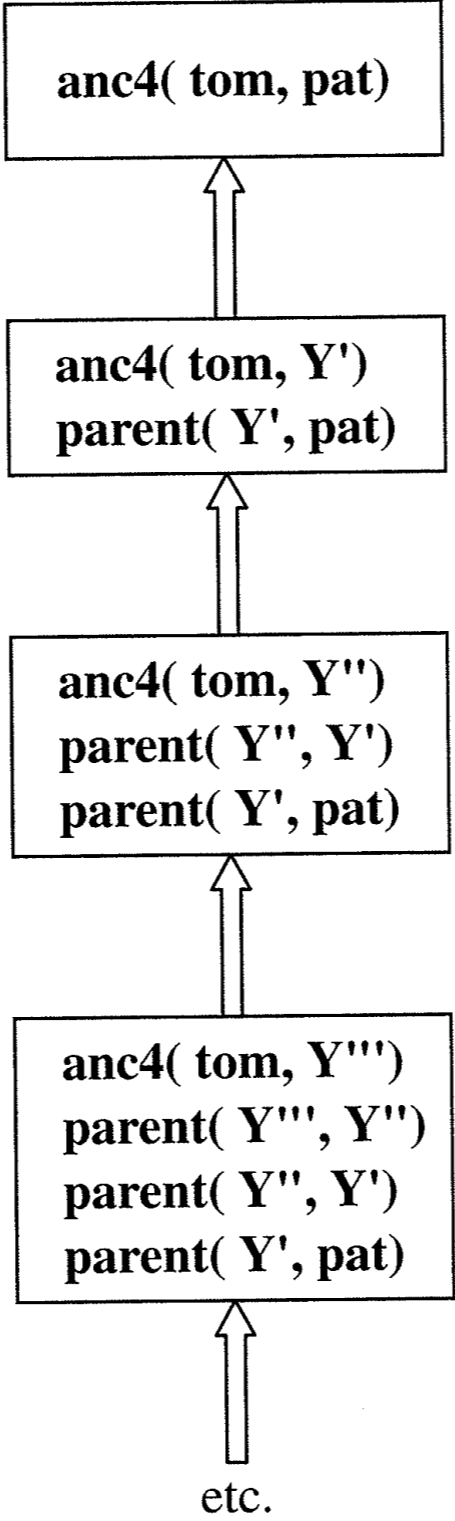


Figure 2.15 Infinite execution trace to satisfy the goal `anc4(tom, pat)`.