

Verification of Dekker's Algorithm

Proof of mutual exclusion

Algorithm 4.2: Dekker's algorithm

boolean wantp \leftarrow false, wantq \leftarrow false
integer turn \leftarrow 1

p

q

loop forever

p1: non-critical section
p2: wantp \leftarrow true
p3: while wantq
p4: if turn = 2
p5: wantp \leftarrow false
p6: await turn = 1
p7: wantp \leftarrow true
p8: critical section
p9: turn \leftarrow 2
p10: wantp \leftarrow false

loop forever

q1: non-critical section
q2: wantq \leftarrow true
q3: while wantp
q4: if turn = 1
q5: wantq \leftarrow false
q6: await turn = 2
q7: wantq \leftarrow true
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Proof of mutual exclusion

Algorithm 4.2: Dekker's algorithm	
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Lemma 4.10

$$(4.2) \text{ turn} = 1 \vee \text{turn} = 2$$

$$(4.3) p3..5 \vee p8..10 \equiv \text{want } p$$

$$(4.4) q3..5 \vee q8..10 \equiv \text{want } q$$

Proof of mutual exclusion

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Lemma 4.10

$$(4.2) \text{ turn} = 1 \vee \text{turn} = 2$$

$$(4.3) p_{3..5} \vee p_{8..10} \equiv \text{want } p$$

$$(4.4) q_{3..5} \vee q_{8..10} \equiv \text{want } q$$

It is an exercise for the student to prove these lemmas and to use them to prove that Dekker's algorithm satisfies mutual exclusion.

Proof of starvation-free

Assumptions:

```
loop forever
p1:   non-critical section
p2:   wantp ← true
p3:   while wantq
p4:     if turn = 2
p5:       wantp ← false
p6:       await turn = 1
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p10:  wantp ← false
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Proof of starvation-free

Assumptions:

1. All assignment statements progress.

```
loop forever
p1:  non-critical section
→ p2:  wantp ← true
p3:  while wantq
p4:    if turn = 2
→ p5:    wantp ← false
p6:    await turn = 1
→ p7:    wantp ← true
p8:  critical section
→ p9:  turn ← 2
→ p10: wantp ← false
```

Proof of starvation-free

Assumptions:

1. All assignment statements progress.
2. $\square (p8 \Rightarrow \diamond p9)$

loop forever

p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

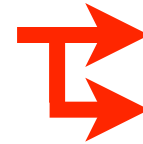
p6: await turn = 1

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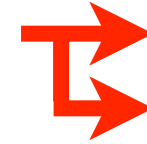


Proof of starvation-free

Assumptions:

1. All assignment statements progress.
2. $\square (p8 \Rightarrow \diamond p9)$
3. $q1 \Rightarrow \diamond q2$

Progress assumption (PA), $\square \diamond \neg q1$
($\diamond \square q1$, exercise for the student.)



```
loop forever
q1:  non-critical section
q2:  wantq ← true
q3:  while wantp
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4. $p4 \wedge \square (turn = 2) \Rightarrow \diamond p5$

loop forever

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p2: wantp \leftarrow true

p3: while wantq

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p5: wantp \leftarrow false

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4. $p4 \wedge \square (turn = 2) \Rightarrow \diamond p5$

Note: Neither of the following is true

$p4 \wedge (turn = 2) \Rightarrow \diamond p5$

$p4 \wedge \neg(turn = 2) \Rightarrow \diamond p3$

because of possible inter leavings outside the CS of q .

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p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

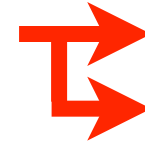
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$p4 \wedge (turn = 2) \Rightarrow \Diamond p5$

$p4 \wedge \neg(turn = 2) \Rightarrow \Diamond p3$

because of possible inter leavings outside the CS of q .

5. The progress proof rule: $\Diamond \Box A \wedge (\Box A \Rightarrow \Diamond B) \Rightarrow \Diamond B$

loop forever

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p2: wantp \leftarrow true

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5. The progress proof rule: $\diamond \square A \wedge (\square A \Rightarrow \diamond B) \Rightarrow \diamond B$

6. All computations are weakly fair.

loop forever

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6. All computations are weakly fair.

And similarly for $p, q := q, p$.

loop forever

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Proof of starvation-free

Lemma 4.11

If p insists on entering, eventually q will let it.

(L 4.11) $\Box want p \wedge \Box turn = 1 \Rightarrow \Diamond \Box \neg want q$

loop forever

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q1:   non-critical section
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$q1$ in initial state



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Proof: Assume conjuncts of antecedent.

$q1$ in initial state

$\Rightarrow \langle 3. q1 \Rightarrow \Diamond q2 \rangle$



loop forever

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$\Diamond q2$

$\Rightarrow \langle 1. \text{Assignments progress.} \rangle$



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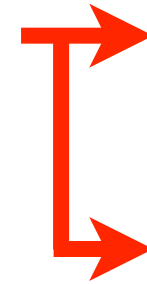
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$\Diamond q3$

$\Rightarrow \langle \text{Conjunct } \Box want p \rangle$

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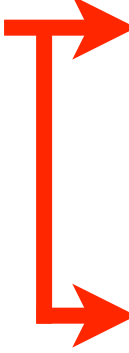
$\Diamond q4$

$\Rightarrow \langle \text{Conjunct } \Box turn = 1 \rangle$

$\Diamond q5$

$\Rightarrow \langle 1. \text{Assignments progress.} \rangle$

loop forever



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$\Diamond q4$

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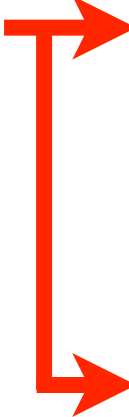
$\Diamond q5$

$\Rightarrow \langle 1. \text{Assignments progress.} \rangle$

$\Diamond q6$

$\Rightarrow \langle \text{Conjunct } \Box turn = 1 \rangle$

loop forever



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$\Diamond \Box \neg(q3\dots5 \vee q8\dots10)$

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$\Diamond \Box \neg(q3\dots5 \vee q8\dots10)$

$\Rightarrow \langle (4.4) q3\dots5 \vee q8\dots10 \equiv want q \rangle$

loop forever

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$\Diamond \Box \neg want q \quad //$

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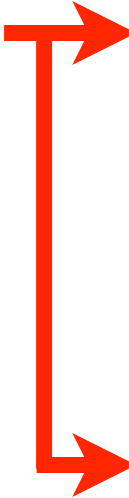
Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

loop forever

p1: non-critical section
p2: wantp \leftarrow true
p3: while wantq
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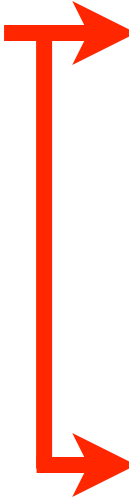
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Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

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loop forever
p1:   non-critical section
p2:   wantp ← true
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Proof of starvation-free

Theorem 4.12

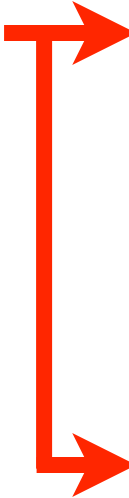
Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$$\neg(p2 \Rightarrow \diamond p8)$$

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Proof of starvation-free

Theorem 4.12

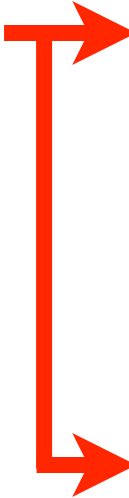
Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$$\begin{aligned} & \neg(p2 \Rightarrow \diamond p8) \\ = & \langle (3.59) p \Rightarrow q \equiv \neg p \vee q \rangle \end{aligned}$$

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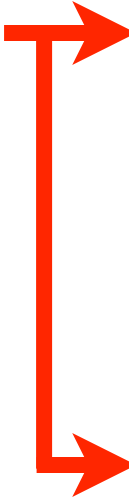
Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$$\begin{aligned} & \neg(p2 \Rightarrow \diamond p8) \\ = & \langle (3.59) p \Rightarrow q \equiv \neg p \vee q \rangle \\ & \neg(\neg p2 \vee \diamond p8) \end{aligned}$$

loop forever

p1: non-critical section
p2: wantp ← true
p3: while wantq
p4: if turn = 2
p5: wantp ← false
p6: await turn = 1
p7: wantp ← true
p8: critical section
p9: turn ← 2
p10: wantp ← false



Proof of starvation-free

Theorem 4.12

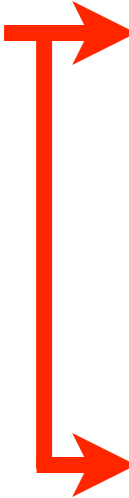
Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$$\begin{aligned} & \neg(p2 \Rightarrow \diamond p8) \\ = & \langle (3.59) p \Rightarrow q \equiv \neg p \vee q \rangle \\ & \neg(\neg p2 \vee \diamond p8) \\ = & \langle \text{De Morgan and double negation} \rangle \end{aligned}$$

loop forever

p1: non-critical section
p2: wantp ← true
p3: while wantq
p4: if turn = 2
p5: wantp ← false
p6: await turn = 1
p7: wantp ← true
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Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

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loop forever

p1: non-critical section
p2: wantp ← true
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Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$$\begin{aligned} & \neg(p2 \Rightarrow \diamond p8) \\ = & \langle(3.59) p \Rightarrow q \equiv \neg p \vee q\rangle \\ & \neg(\neg p2 \vee \diamond p8) \\ = & \langle\text{De Morgan and double negation}\rangle \\ & p2 \wedge \neg \diamond p8 \\ \Rightarrow & \langle\text{Lemma A: } p2 \wedge \neg \diamond p8 \Rightarrow p3, p4 \text{ forever}\rangle \end{aligned}$$

loop forever

p1:	non-critical section
p2:	wantp \leftarrow true
p3:	while wantq
p4:	if turn = 2
p5:	wantp \leftarrow false
p6:	await turn = 1
p7:	wantp \leftarrow true
p8:	critical section
p9:	turn \leftarrow 2
p10:	wantp \leftarrow false

Proof of starvation-free

Theorem 4.12

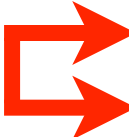
Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$$\begin{aligned} & \neg(p2 \Rightarrow \diamond p8) \\ = & \langle (3.59) p \Rightarrow q \equiv \neg p \vee q \rangle \\ & \neg(\neg p2 \vee \diamond p8) \\ = & \langle \text{De Morgan and double negation} \rangle \\ & p2 \wedge \neg \diamond p8 \\ \Rightarrow & \langle \text{Lemma A: } p2 \wedge \neg \diamond p8 \Rightarrow p3, p4 \text{ forever} \rangle \\ & p3, p4 \text{ forever} \end{aligned}$$

loop forever

p1: non-critical section
p2: wantp ← true
p3: while wantq
p4: if turn = 2
p5: wantp ← false
p6: await turn = 1
p7: wantp ← true
p8: critical section
p9: turn ← 2
p10: wantp ← false



Proof of starvation-free

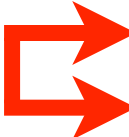
Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$p3, p4$ forever

```
loop forever
p1:   non-critical section
p2:   wantp ← true
p3:   while wantq
p4:       if turn = 2
p5:           wantp ← false
p6:           await turn = 1
p7:           wantp ← true
p8:   critical section
p9:   turn ← 2
p10:  wantp ← false
```



Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$p3, p4$ forever

= $\langle (4.3) p3..5 \vee p8..10 \equiv want p,$

code inspection \rangle

loop forever

p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

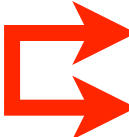
p6: await turn = 1

p7: wantp \leftarrow true

p8: critical section

p9: turn \leftarrow 2

p10: wantp \leftarrow false



Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

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code inspection \rangle

$\square want p \wedge \square turn = 1$

loop forever

p1: non-critical section

p2: $wantp \leftarrow true$

p3: while $wantq$

p4: if $turn = 2$

p5: $wantp \leftarrow false$

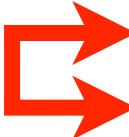
p6: $await turn = 1$

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Proof of starvation-free

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= $\langle (4.3) p3..5 \vee p8..10 \equiv want p,$
code inspection \rangle

$\square want p \wedge \square turn = 1$

$\Rightarrow \langle (L 4.11)$

$\square want p \wedge \square turn = 1 \Rightarrow \diamond \square \neg want q \rangle$

loop forever

p1: non-critical section

p2: $wantp \leftarrow true$

p3: while $wantq$

p4: if $turn = 2$

p5: $wantp \leftarrow false$

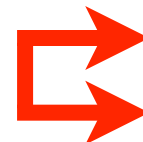
p6: $await turn = 1$

p7: $wantp \leftarrow true$

p8: critical section

p9: $turn \leftarrow 2$

p10: $wantp \leftarrow false$



Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$p3, p4$ forever

= $\langle (4.3) p3..5 \vee p8..10 \equiv want p,$
code inspection \rangle

$\square want p \wedge \square turn = 1$

$\Rightarrow \langle (L 4.11)$

$\square want p \wedge \square turn = 1 \Rightarrow \diamond \square \neg want q \rangle$

$\diamond \square \neg want q$

loop forever

p1: non-critical section

p2: $wantp \leftarrow true$

p3: while $wantq$

p4: if $turn = 2$

p5: $wantp \leftarrow false$

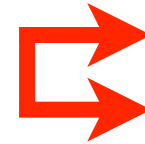
p6: $await turn = 1$

p7: $wantp \leftarrow true$

p8: critical section

p9: $turn \leftarrow 2$

p10: $wantp \leftarrow false$



Proof of starvation-free

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code inspection \rangle

$\square want p \wedge \square turn = 1$

$\Rightarrow \langle (L 4.11)$

$\square want p \wedge \square turn = 1 \Rightarrow \diamond \square \neg want q \rangle$

$\diamond \square \neg want q$

$\Rightarrow \langle p3: \text{while want} q \rangle$

loop forever

p1: non-critical section

p2: $wantp \leftarrow true$

p3: while $wantq$

p4: if $turn = 2$

p5: $wantp \leftarrow false$

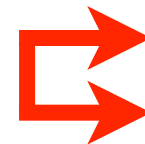
p6: $await turn = 1$

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p8: critical section

p9: $turn \leftarrow 2$

p10: $wantp \leftarrow false$



Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

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= $\langle (4.3) p3..5 \vee p8..10 \equiv want p,$
code inspection \rangle

$\square want p \wedge \square turn = 1$

$\Rightarrow \langle (L 4.11)$

$\square want p \wedge \square turn = 1 \Rightarrow \diamond \square \neg want q \rangle$

$\diamond \square \neg want q$

$\Rightarrow \langle p3: \text{while } wantq \rangle$

$\neg(p3, p4 \text{ forever})$

loop forever

p1: non-critical section

p2: $wantp \leftarrow true$

p3: while $wantq$

p4: if $turn = 2$

p5: $wantp \leftarrow false$

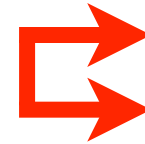
p6: $await \text{ turn} = 1$

p7: $wantp \leftarrow true$

p8: critical section

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Proof of starvation-free

Theorem 4.12

Process p does not starve: $p2 \Rightarrow \diamond p8$

Proof: By contradiction.

$p3, p4$ forever

= $\langle (4.3) p3..5 \vee p8..10 \equiv want p,$
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$\square want p \wedge \square turn = 1$

$\Rightarrow \langle (L 4.11)$

$\square want p \wedge \square turn = 1 \Rightarrow \diamond \square \neg want q \rangle$

$\diamond \square \neg want q$

$\Rightarrow \langle p3: \text{while } wantq \rangle$

$\neg(p3, p4 \text{ forever})$

Contradiction //

loop forever

p1: non-critical section

p2: $wantp \leftarrow true$

p3: while $wantq$

p4: if $turn = 2$

p5: $wantp \leftarrow false$

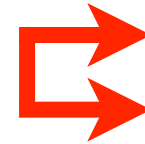
p6: await $turn = 1$

p7: $wantp \leftarrow true$

p8: critical section

p9: $turn \leftarrow 2$

p10: $wantp \leftarrow false$



Proof of starvation-free


Lemma A


$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever


Proof:


loop forever


p1: non-critical section


 p2: wantp ← true


 p3: while wantq

 p4: if turn = 2

 p5: wantp ← false

 p6: await turn = 1

 p7: wantp ← true

 p8: critical section

p9: turn ← 2

p10: wantp ← false

Proof of starvation-free

Lemma A








$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

First

$p2 \wedge \neg \diamond p8$

loop forever

p1:	non-critical section
 p2:	wantp ← true
 p3:	while wantq
 p4:	if turn = 2
 p5:	wantp ← false
 p6:	await turn = 1
 p7:	wantp ← true
 p8:	critical section
p9:	turn ← 2
p10:	wantp ← false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4 \text{ forever}$

Proof:

First

$p2 \wedge \neg \diamond p8$

$\Rightarrow \langle \text{Lemma B, } p2 \wedge \neg \diamond p8 \Rightarrow \neg \square \text{turn} = 2 \rangle$

loop forever

p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

p6: await turn = 1

p7: wantp \leftarrow true

p8: critical section

p9: turn \leftarrow 2

p10: wantp \leftarrow false



Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4 \text{ forever}$

Proof:

First

$p2 \wedge \neg \diamond p8$
 $\Rightarrow \langle \text{Lemma B, } p2 \wedge \neg \diamond p8 \Rightarrow \neg \square \textit{turn} = 2 \rangle$
 $\neg \square \textit{turn} = 2$

loop forever

p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

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p5: wantp \leftarrow false

p6: await turn = 1

p7: wantp \leftarrow true

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p10: wantp \leftarrow false



Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever


Proof:


First


$$\begin{aligned} & p2 \wedge \neg \diamond p8 \\ \Rightarrow & \langle \text{Lemma B, } p2 \wedge \neg \diamond p8 \Rightarrow \neg \square \text{turn} = 2 \rangle \\ & \neg \square \text{turn} = 2 \\ = & \langle (51) \text{ Dual of } \square, \neg \square p \equiv \diamond \neg p \rangle \end{aligned}$$


loop forever


p1: non-critical section


 p2: wantp ← true


 p3: while wantq

 p4: if turn = 2

 p5: wantp ← false

 p6: await turn = 1

 p7: wantp ← true

 p8: critical section

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Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4 \text{ forever}$

Proof:

First

$$\begin{aligned} & p2 \wedge \neg \diamond p8 \\ \Rightarrow & \langle \text{Lemma B, } p2 \wedge \neg \diamond p8 \Rightarrow \neg \square \text{turn} = 2 \rangle \\ & \neg \square \text{turn} = 2 \\ = & \langle (51) \text{ Dual of } \square, \neg \square p \equiv \diamond \neg p \rangle \\ & \diamond \neg \text{turn} = 2 \end{aligned}$$

loop forever

p1:	non-critical section
p2:	wantp \leftarrow true
p3:	while wantq
p4:	if turn = 2
p5:	wantp \leftarrow false
p6:	await turn = 1
p7:	wantp \leftarrow true
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p10:	wantp \leftarrow false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4 \text{ forever}$

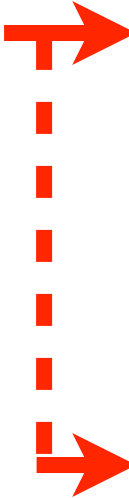
Proof:

First

$p2 \wedge \neg \diamond p8$
 $\Rightarrow \langle \text{Lemma B, } p2 \wedge \neg \diamond p8 \Rightarrow \neg \square \text{turn} = 2 \rangle$
 $\neg \square \text{turn} = 2$
 $= \langle (51) \text{ Dual of } \square, \neg \square p \equiv \diamond \neg p \rangle$
 $\diamond \neg \text{turn} = 2$
 $\Rightarrow \langle (4.2) \text{turn} = 1 \vee \text{turn} = 2 \rangle$

loop forever

p1: non-critical section
p2: wantp \leftarrow true
p3: while wantq
p4: if turn = 2
p5: wantp \leftarrow false
p6: await turn = 1
p7: wantp \leftarrow true
p8: critical section
p9: turn \leftarrow 2
p10: wantp \leftarrow false



Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

First

$p2 \wedge \neg \diamond p8$
 $\Rightarrow \langle \text{Lemma B, } p2 \wedge \neg \diamond p8 \Rightarrow \neg \square \text{turn} = 2 \rangle$
 $\neg \square \text{turn} = 2$
 $= \langle (51) \text{ Dual of } \square, \neg \square p \equiv \diamond \neg p \rangle$
 $\diamond \neg \text{turn} = 2$
 $\Rightarrow \langle (4.2) \text{turn} = 1 \vee \text{turn} = 2 \rangle$
 $\diamond \text{turn} = 1$

loop forever

p1: non-critical section
p2: wantp ← true
p3: while wantq
p4: if turn = 2
p5: wantp ← false
p6: await turn = 1
p7: wantp ← true
p8: critical section
p9: turn ← 2
p10: wantp ← false

Proof of starvation-free

Lemma A








$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

Second

$p2 \wedge \neg \diamond p8$

loop forever

p1:	non-critical section
 p2:	wantp ← true
 p3:	while wantq
 p4:	if turn = 2
 p5:	wantp ← false
 p6:	await turn = 1
 p7:	wantp ← true
 p8:	critical section
p9:	turn ← 2
p10:	wantp ← false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever









Proof:

Second

$p2 \wedge \neg \diamond p8$

$\Rightarrow \langle p9 \text{ follows } p8 \rangle$

loop forever

p1:	non-critical section
 p2:	wantp ← true
 p3:	while wantq
 p4:	if turn = 2
 p5:	wantp ← false
 p6:	await turn = 1
 p7:	wantp ← true
 p8:	critical section
 p9:	turn ← 2
p10:	wantp ← false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:









Second

$p2 \wedge \neg \diamond p8$

$\Rightarrow \langle p9 \text{ follows } p8 \rangle$

$p9$ never executes

loop forever

p1:	non-critical section
 p2:	wantp ← true
 p3:	while wantq
 p4:	if turn = 2
 p5:	wantp ← false
 p6:	await turn = 1
 p7:	wantp ← true
 p8:	critical section
 p9:	turn ← 2
p10:	wantp ← false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

Second









$p2 \wedge \neg \diamond p8$

$\Rightarrow \langle p9 \text{ follows } p8 \rangle$

$p9$ never executes

$\Rightarrow \langle \text{Only } p9 \text{ sets turn to 2} \rangle$

loop forever

p1:	non-critical section
 p2:	wantp \leftarrow true
 p3:	while wantq
 p4:	if turn = 2
 p5:	wantp \leftarrow false
 p6:	await turn = 1
 p7:	wantp \leftarrow true
 p8:	critical section
 p9:	turn \leftarrow 2
p10:	wantp \leftarrow false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

Second

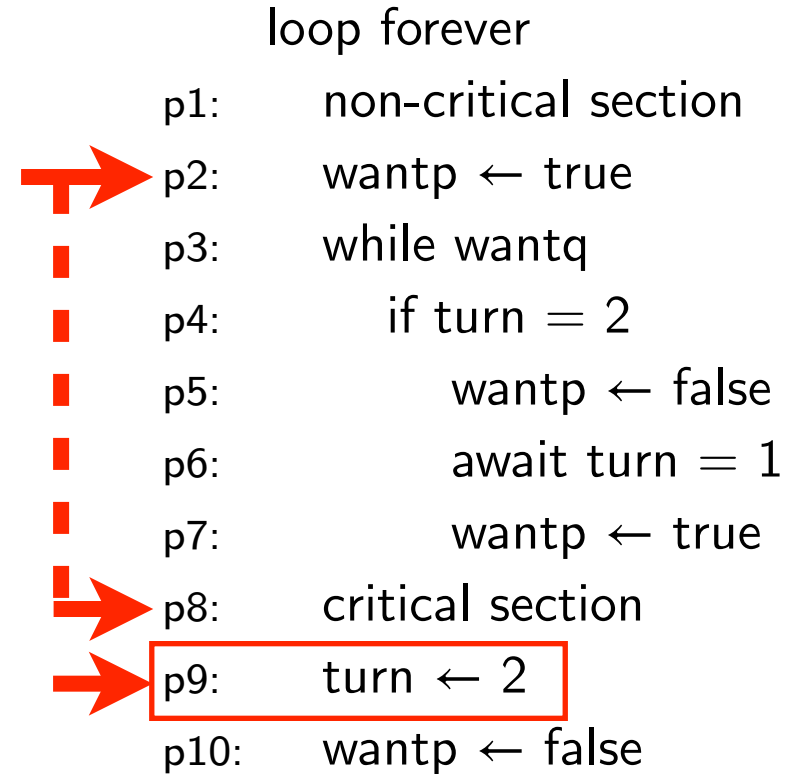
$p2 \wedge \neg \diamond p8$

$\Rightarrow \langle p9 \text{ follows } p8 \rangle$

$p9$ never executes

$\Rightarrow \langle \text{Only } p9 \text{ sets turn to 2} \rangle$

turn is never set to 2



Proof of starvation-free








Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

First, $p2 \wedge \neg \diamond p8 \Rightarrow \diamond turn = 1$

loop forever

p1:	non-critical section
 p2:	wantp \leftarrow true
 p3:	while wantq
 p4:	if turn = 2
 p5:	wantp \leftarrow false
 p6:	await turn = 1
 p7:	wantp \leftarrow true
 p8:	critical section
p9:	turn \leftarrow 2
p10:	wantp \leftarrow false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever


Proof:


First, $p2 \wedge \neg \diamond p8 \Rightarrow \diamond turn = 1$


Second, $p2 \wedge \neg \diamond p8 \Rightarrow$ turn is never set to 2


loop forever


p1: non-critical section


 p2: wantp ← true


 p3: while wantq

 p4: if turn = 2

 p5: wantp ← false

 p6: await turn = 1

 p7: wantp ← true

 p8: critical section

p9: turn ← 2

p10: wantp ← false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:


First, $p2 \wedge \neg \diamond p8 \Rightarrow \diamond turn = 1$


Second, $p2 \wedge \neg \diamond p8 \Rightarrow$ turn is never set to 2


Therefore, $\diamond \square turn = 1$


loop forever


p1: non-critical section


 p2: wantp ← true


 p3: while wantq

 p4: if turn = 2

 p5: wantp ← false

 p6: await turn = 1

 p7: wantp ← true

 p8: critical section

p9: turn ← 2

p10: wantp ← false

Proof of starvation-free

Lemma A

$p2 \wedge \neg \diamond p8 \Rightarrow p3, p4$ forever

Proof:

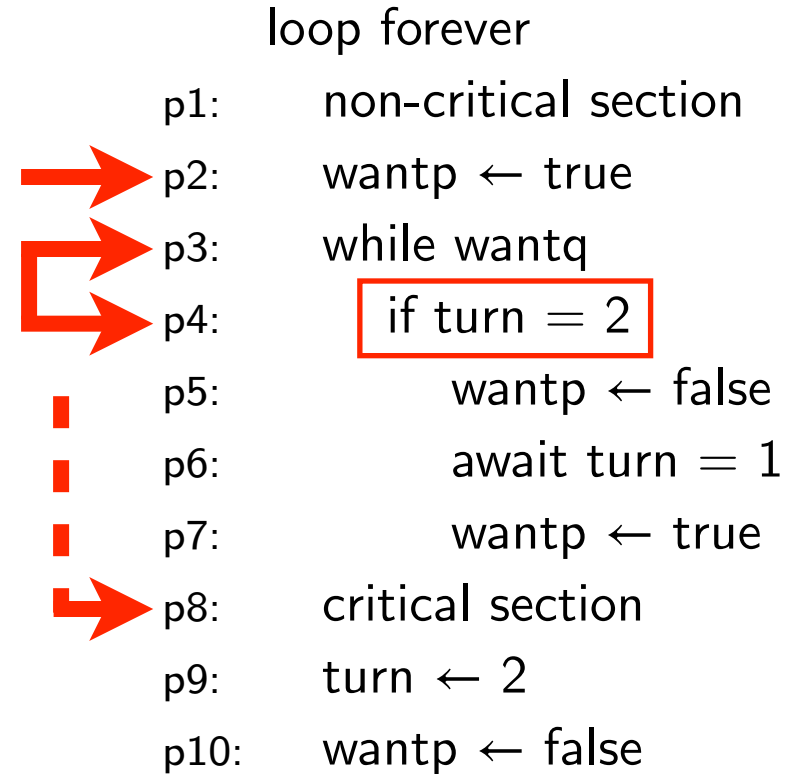
First, $p2 \wedge \neg \diamond p8 \Rightarrow \diamond turn = 1$

Second, $p2 \wedge \neg \diamond p8 \Rightarrow$ turn is never set to 2

Therefore, $\diamond \square turn = 1$

So, the only way p cannot reach $p8$ is if

$p3, p4$ forever with $turn = 1$ //



Proof of starvation-free


Lemma B

$p2 \wedge \neg \diamond p8 \Rightarrow \neg \square turn = 2$

Proof: By contradiction

loop forever

p1: non-critical section

 p2: wantp ← true

p3: while wantq

p4: if turn = 2

p5: wantp ← false

p6: await turn = 1

p7: wantp ← true

p8: critical section

p9: turn ← 2

p10: wantp ← false

Proof of starvation-free

Lemma B


$p2 \wedge \neg \diamond p8 \Rightarrow \neg \square turn = 2$

Proof: By contradiction

$\square turn = 2$

loop forever

p1: non-critical section

 p2: wantp ← true

p3: while wantq

p4: if turn = 2

p5: wantp ← false

p6: await turn = 1

p7: wantp ← true

p8: critical section

p9: turn ← 2

p10: wantp ← false

Proof of starvation-free

Lemma B

$$p2 \wedge \neg \diamond p8 \Rightarrow \neg \square turn = 2$$


Proof: By contradiction

$$\square turn = 2$$

\Rightarrow \langle Progress, code inspection \rangle

loop forever

p1: non-critical section

 p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

p6: await turn = 1

p7: wantp \leftarrow true

p8: critical section

p9: turn \leftarrow 2

p10: wantp \leftarrow false

Proof of starvation-free

Lemma B

$$p2 \wedge \neg \diamond p8 \Rightarrow \neg \square turn = 2$$

Proof: By contradiction

$$\square turn = 2$$

\Rightarrow \langle Progress, code inspection \rangle

loop forever

p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

p6: await turn = 1

p7: wantp \leftarrow true

p8: critical section

p9: turn \leftarrow 2

p10: wantp \leftarrow false

Proof of starvation-free

Lemma B

$$p2 \wedge \neg \diamond p8 \Rightarrow \neg \square turn = 2$$

Proof: By contradiction

$$\square turn = 2$$

\Rightarrow \langle Progress, code inspection \rangle

loop forever

p1: non-critical section

p2: wantp \leftarrow true

p3: while wantq

p4: if turn = 2

p5: wantp \leftarrow false

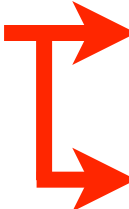
p6: await turn = 1

p7: wantp \leftarrow true

p8: critical section

p9: turn \leftarrow 2

p10: wantp \leftarrow false



Proof of starvation-free

Lemma B

$$p2 \wedge \neg \diamond p8 \Rightarrow \neg \square turn = 2$$

Proof: By contradiction

$$\square turn = 2$$

\Rightarrow \langle Progress, code inspection \rangle

loop forever

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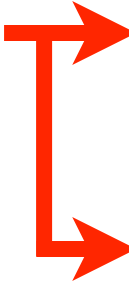
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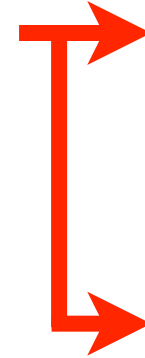
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$$\diamond q9$$

\Rightarrow \langle q9: $turn \leftarrow 1$ \rangle

$$\diamond turn = 1$$

Contradicts $\square turn = 2$ //

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