

The derivation for a sequence of two assignment statements when the unknown expression is in the second assignment is more complex than when the expression is in the first assignment. Because the weakest precondition is computed from right-to-left, you must consider the textual substitution that may occur in the expression when computing the weakest precondition of the first assignment.

This example assumes x and y are two Fibonacci numbers and replaces them with the next pair of Fibonacci numbers. Unknown expression F is in the second assignment and has a textual substitution in the derivation. X and Y are rigid variables.

$$\{x = X \wedge y = Y\} y := E ; x := F \{x = Y \wedge y = X + Y\}$$

The derivation uses (4.4) deduction with (p.4) the definition of the Hoare triple: $\{Q\} S \{R\} \equiv Q \Rightarrow wp.S.R$

Here is the derivation beginning with the consequent of the definition.

$$\begin{aligned} & wp.(y := E ; x := F).(x = Y \wedge y = X + Y) \\ = & \langle \text{(p.14) Definition, Composition} \rangle \\ & wp.(y := E).wp.(x := F).(x = Y \wedge y = X + Y) \\ = & \langle \text{(p.18) and textual substitution} \rangle \\ & wp.(y := E).(F = Y \wedge y = X + Y) \\ = & \langle \text{(p.18) and textual substitution} \rangle \\ & F_E^y = Y \wedge E = X + Y \\ = & \langle \text{Assume conjuncts } x = X \text{ and } y = Y \rangle \\ & F_E^y = y \wedge E = x + y \\ = & \langle \text{(3.84a) Substitution } (e = f) \wedge E_e^z \equiv (e = f) \wedge E_f^z \rangle \\ & F_{x+y}^y = y \wedge E = x + y \end{aligned}$$

The derivation shows that $E = x + y$ but what is the expression F ? Recall that F_{x+y}^y is an abbreviation for the textual substitution $F[y := x + y]$. The derivation shows that F is an expression such that if you make the textual substitution $[y := x + y]$ you get y . Working backward, F must be the expression

$$F = y - x$$

because

$$F_{x+y}^y = F[y := x + y] = (y - x)[y := x + y] = x + y - x = y$$

So, the program is

$$y := x + y ; x := y - x$$

For example, if the initial state is the pair of Fibonacci numbers $(x, 3), (y, 5)$ and you execute the program then the final state is $(x, 5), (y, 8)$, which is the next pair of Fibonacci numbers.